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The density of states in quasiperiodic photonic crystals

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Abstract

The distribution of the local density of states (LDOS) in quasiperiodic photonic crystals (QPCs) and the total density of states (DOS) are studied. The distribution of the LDOS shows that it has the same rotational symmetry as the sample. Furthermore, the regions of lowest LDOS and the change of the minimal values of the LDOS with the alteration of the dielectric volume fractions are identified. Meanwhile, the coincidence of gaps displayed in transmission and DOS spectra indicates that QPCs possess absolute gaps for TM polarized modes, the E field is along the axis of the cylinders and the H field is in the 2D plane.

Photonic band-gap (PBG) material, since first proposed by Yablonovitch [1] and John [2], has attracted considerable attention [3–5]. A great deal of theoretical and experimental effort [6, 7] has been devoted to this field. One of the early motivations for the study of photonic crystals is their potential for the control of electromagnetic radiation from quantized sources [1]. The suppression of spontaneous emission, as we know, is important in keeping a system in excited states and can result in the reduction of noise in optoelectronic devices. However, spontaneous emission is not an intrinsic property of molecules. It relates closely to the local density of states (LDOS) [8, 9]. So, having a clear picture of the distribution of LDOS is important for the use of photonic crystals. At the same time, the distribution of the LDOS relates closely to the structure of the sample. It may be an effective means in the analysis of structure's symmetry. Meanwhile, in most cases the photonic gap is indicated by corresponding transmission or reflection spectra. Low transmission regions are generally considered as the gap. However, the dependence of transmission on the incident direction makes this criterion ambiguous. Therefore, the density of states (DOS) is more accurate than transmittance in deciding the gaps of photonic materials.

Various methods have been employed in the calculation of DOS [10–13]. However, little has been reported on the distribution of LDOS in a finite size photonic crystal. Recently, Asatryan *et al* [14] have extended the exact formalism of multipole expansions to construct a Green function $G(r, r_s, \omega)$ of finite-sized two-dimensional photonic crystals and calculated the LDOS and DOS of it. They obtained a clear picture about the distribution of the LDOS inside periodic photonic crystals. Meanwhile, owing to the anisotropy, the gap shown in the transmission spectrum is quite different from that in the DOS spectrum; quasiperiodic photonic

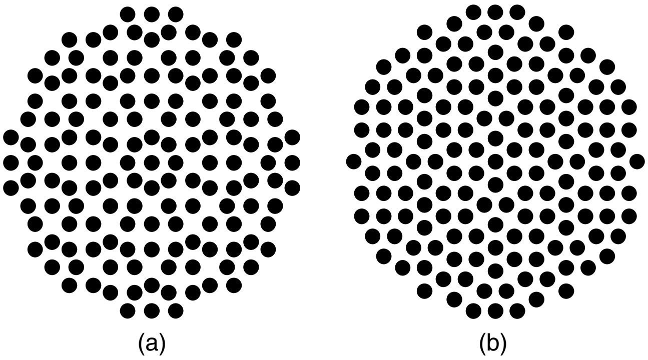

Figure 1. (a) Octagonal and (b) dodecagonal quasicrystal.

crystals (QPCs) [10, 15–18] are quite different in many aspects from periodic ones. However, LDOS and the relation of the gaps shown in transmission and DOS spectra of QPCs have not been studied. They may have some interesting features. In this paper, we calculate the LDOS and DOS as well as transmission spectra of octagonal and dodecagonal QPCs using the multiple-scattering methods [19]. We find that the distribution of LDOS shows the same rotational symmetry as the sample. Meanwhile, the regions of the lowest LDOS are recognized. In addition, the gaps shown in transmission and DOS spectra are almost coincident, especially for dodecagonal quasicrystal. From this we can affirm that these structures possess absolute gaps for TM polarized modes whose E field is along the axis of the cylinders and H field is in the 2D plane.

The octagonal and dodecagonal QPCs are shown in figure 1. They are formed by placing dielectric cylinders with circular cross sections in the vertices of two-dimensional octagonal and dodecagonal quasiperiodic lattices [16–18], respectively. The octagonal quasiperiodic pattern is tiled by squares and rhombuses (with an acute angle of 45°) of equal side length $a = 11.0$ mm. The dielectric constants of the cylinders and their background are 8.9 and 1.04, respectively. The dodecagonal quasiperiodic pattern is tiled by squares and regular triangles of the same side length $a = 5.0$ mm. The dielectric constant of the cylinders in dodecagonal QPCs is 8.5 and that of the background is the same as the octagonal one.

In our simulations of transmission spectra, we assume that the incident beam passes through a slit with a width of $w = 4a$, which is positioned in front of the sample with a distance of $5a$ between the centre of the slit and the surface of the sample, and then illuminates the sample. In this case the incident field can be obtained from the Kirchhoff integral formula. In two dimensions, for a plane wave $\exp(\mathrm{i}kx)$ incident from $x < 0$, the diffracted wave in the region $x > 0$ arising from a slit centred at the origin with an opening of width w in the y direction is given by

$$u_{\mathrm{inc}}(x, y) = \left(\frac{k_0}{4}\right) \int_{-w/2}^{w/2} \mathrm{d}y' \left[H_0(k_0\rho') + \mathrm{i}\frac{x}{\rho'} H_1(k_0\rho') \right] \quad (1)$$

where $\rho' = \sqrt{x^2 + (y - y')^2}$, and H_m is a Hankel function of the first kind. A generalized transmission coefficient is defined as the ratio of energy flux to that of the incident wave at $\theta = 0$,

$$T = \left|1 + \sqrt{(2\pi/k_0 w)}\mathrm{e}^{\mathrm{i}\pi/4} f_{\mathrm{s}}(0)\right|^2 \quad (2)$$

where $f_{\mathrm{s}}(0)$ is the total amplitude at $\theta = 0$ in the far field, which can be calculated by a multiple-scattering method.

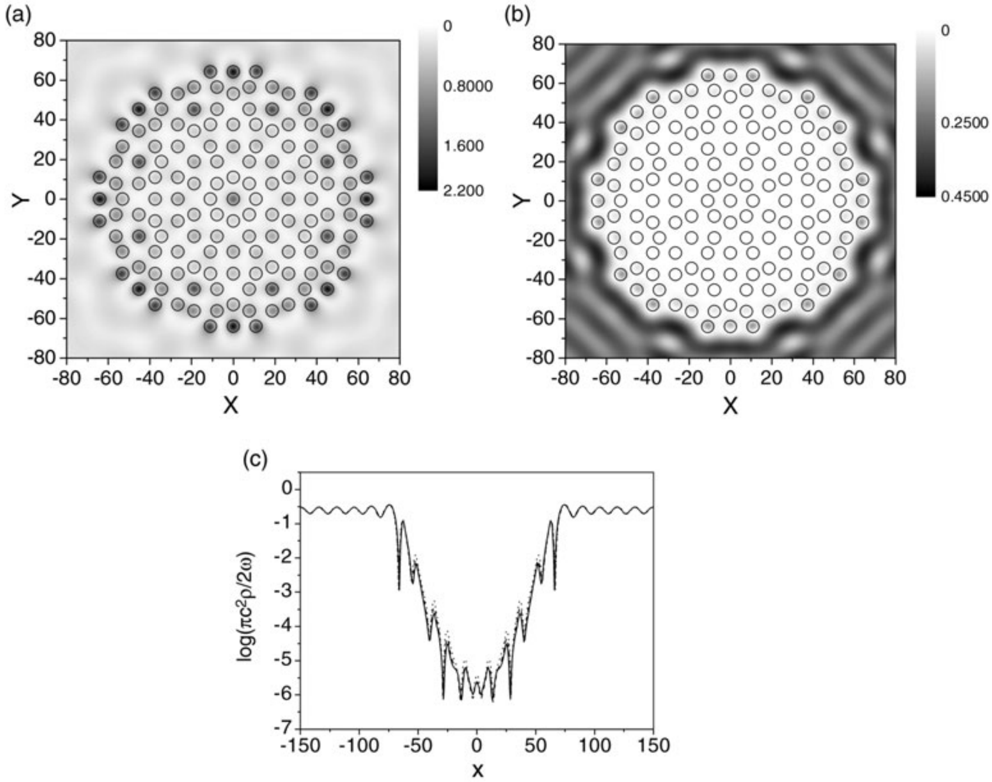

Figure 2. Distributions of $\pi c^2\rho/2\omega$ versus position of the two-dimensional octagonal quasiperiodic photonic crystal at two frequencies (a) 7.0 GHz and (b) 10.0 GHz, which locate within the high and low transmission regions, respectively, and (c) the magnitudes of the LDOS at 10 GHz along the x axis of samples with three volume fractions of 14.5% (solid curve), 17.4% (dashed curve) and 20.6% (dotted curve).

In the calculation of LDOS and DOS, the source is an infinite line antenna that is parallel to the axes of the cylinders. The LDOS in the sample can be written as

$$\rho(r,\omega) = -\frac{2\omega}{\pi c^2}\,\mathrm{Im}[G(r,r_s,\omega)] \tag{3}$$

where $G(r, r_s, \omega)$ is the electromagnetic Green function with a source at location r_s and observation point at r. ω is the angular frequency of the electromagnetic wave which is radiated by the line source, and c is the velocity of light in vacuum. The total DOS $\rho(\omega)$ of the two kinds of QPCs is defined to be the weighted average of the LDOS over the basic cell of the QPCs:

$$\rho(\omega) = \frac{1}{s}\int\int \varepsilon(r)\rho(r,\omega)\,\mathrm{d}x\,\mathrm{d}y \tag{4}$$

where s is the area of the basic cell.

The distribution of the LDOS in octagonal QPCs is shown in figure 2. In our sample there are 145 cylinders and dielectric fraction is 14.46%. Figures 2(a) and (b) are depicted at two frequencies, 7.5 and 10.0 GHz, which locate in the band and gap, respectively. From figure 2(a) we can see that the LDOS is regularly modulated by the cylindrical scatterers. It possesses the same rotational symmetry as the sample itself. In addition, the LDOS within the

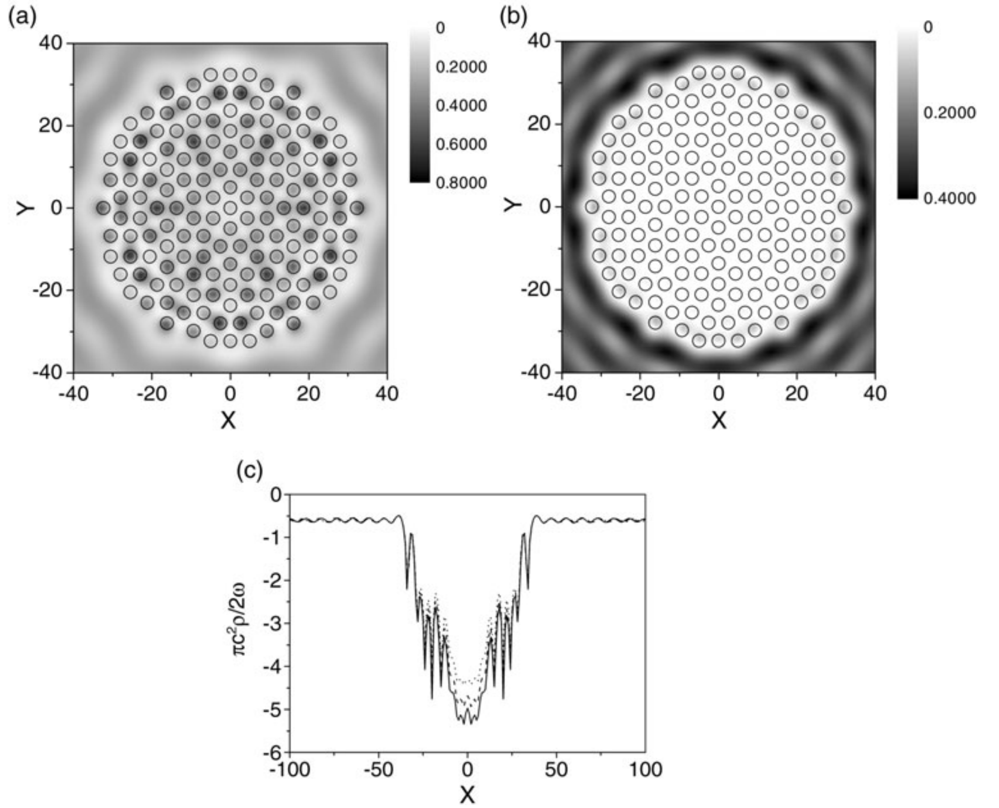

Figure 3. The distribution of $\pi c^2\rho/2\omega$ versus position of the two-dimensional dodecagonal quasiperiodic photonic crystal at two frequencies (a) 12.5 GHz and (b) 17.0 GHz, which locate within the high and low transmission regions, respectively, and (c) the magnitudes of the LDOS at 17.0 GHz along the x axis of samples with three volume fractions of 33.67% (solid curve), 38.01% (dashed curve) and 42.62% (dotted curve).

cylinders is larger than that of the neighbouring background. The reason for this is that the harmonic mode will tend to concentrate its displacement field on the region of high dielectric constant so as to minimize the electromagnetic energy. Meanwhile, figure 2(b) gives the spatial distribution of the LDOS at the frequency 10.0 GHz which locate within the gap. It is seen clearly that the LDOS vanishes in the central area and exists only on the outermost circle of the sample. In order to know quantitatively the distribution of the LDOS in the QPC, we calculate the LDOS along the x axis, (i.e., $y = 0.0$, $x = -150.0$ to $x = +150.0$). Owing to the high rotational symmetry property of the octagonal QPC, it reflects approximately the distribution of the LDOS over the whole sample. The solid line in figure 2(c) shows the results. The LDOS oscillates and decreases exponentially toward the centre. The oscillation peaks always locate at high dielectric regions while the troughs are in low dielectric areas. The minimum of the LDOS occurs in a quite small area which surrounds the central cylinder.

Furthermore, the LDOS of the octagonal QPCs is simulated with different dielectric fractions of 17.4% and 20.6%, where the photonic gap exists, respectively. As shown in figure 2(c), there is little change of minimal values of the LDOS with increasing the volume fraction of scatterers.

Figure 3 is the distribution of the LDOS in the dodecagonal QPC whose volume fraction is 33.67% and the first gap locates from 15.36 to 20.22 GHz. The corresponding frequencies

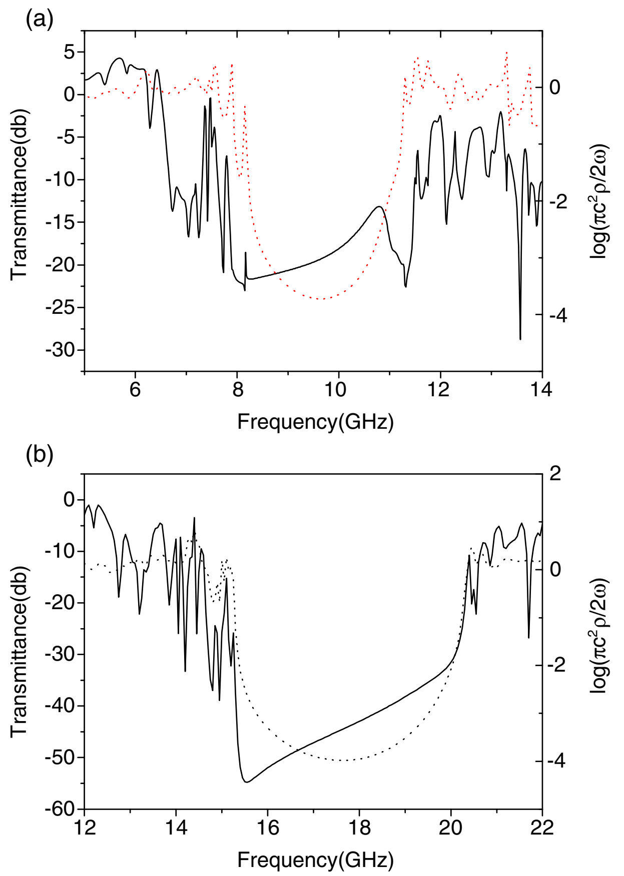

Figure 4. The total DOS (dotted curve) and the transmittance spectrum (solid curve) of octagonal quasiperiodic crystal (a) and dodecagonal quasicrystal (b).

(This figure is in colour only in the electronic version)

of figures 3(a) and (b) are 12.5 and 17.0 GHz, which locate in the band and gap, respectively. The major features of the LDOS here are quite similar to that in octagonal QPCs. First, it possesses 12-fold symmetry. Second, the minimum of the LDOS, as the solid curve shows in figure 3(c), locates in the low dielectric area inside the central hexagon of the QPC. However, compared with the distribution of the LDOS in octagonal QPCs, the response of the LDOS to the volume fraction of the dodecagonal QPC is different. The dashed and dotted curves in figure 3(c) depict the distribution of the LDOS in the dodecagonal QPCs with volume fraction 38.01% and 42.62%, respectively. It shows that the minimal value of the LDOS increases slightly with the increase of volume fraction. The reason for this phenomenon is that the number of the cylinders in the central dodecagon in dodecagonal QPCs is more than that in the central octagon in octagonal QPCs. Increase of the volume fraction makes the average index of refraction inside the central dodecagon in dodecagonal QPCs bigger than inside the central octagon in octagonal QPCs. Therefore, the change of the minimal value of the LDOS with the volume fraction is obvious in the dodecagonal QPCs.

In order to study the band gap structure of QPCs, the DOSs of them are calculated. Figures 4(a) and (b) show the transmittance (solid curve) and the total DOS in logarithmic scale (dot curve) of the octagonal and dodecagonal QPCs, respectively. The gaps displayed in transmission spectra are almost coincident with that shown in DOS spectra. This is different from that of periodic crystals [14], in which the gap shown in the DOS spectrum is much narrower than that revealed in the transmission spectrum. This is attributed to the fact that the periodic crystals are anisotropic, and quasicrystals may be more 'isotropic' than ordinary photonic crystals because quasicrystals possess high rotational symmetries compared to ordinary photonic crystals.

In conclusion, we have studied the distribution of the LDOS and the DOS in octagonal and dodecagonal QPCs. The distribution of the LDOS is modulated by the dielectric constant. It possesses the same rotational symmetry as the QPCs. Meanwhile, the region with minimal values of the LDOS in QPCs and the varying regulation of them with the alteration of dielectric fraction are identified. These are very useful in the application of them. If QPCs are used to suppress spontaneous emission effectively, we should decrease the dielectric fraction of the QPCs as long as the gaps remain. In addition, the coincidence of gaps displayed in the transmission and DOS spectra indicates that QPCs have an absolute gap for TM modes.

Acknowledgments

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